

# Amplitude mode oscillations in pump-probe photoemission spectra from a $d$ -wave superconductor

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Recent developments in the techniques of ultrafast pump-probe photoemission have made possible the search for collective modes in strongly correlated systems out of equilibrium. Including inelastic scattering processes and a retarded interaction, we simulate time- and angle- resolved photoemission spectroscopy (trARPES) to study the amplitude mode of a  $d$ -wave superconductor, a collective mode excited through the nonlinear light-matter coupling to the pump pulse. We find that the amplitude mode oscillations of the  $d$ -wave order parameter occur in phase at a single frequency that is twice the quasi-steady-state maximum gap size after pumping. We comment on the necessary conditions for detecting the amplitude mode in trARPES experiments.

The amplitude mode of the superconducting order parameter, also known as the Higgs mode, is fundamental to superconductivity and arises because of the broken gauge symmetry of the superconducting state. Observing this mode is interesting from the perspective of understanding the collective behavior of a macroscopic quantum state out of equilibrium, especially in strongly correlated systems such as the high-temperature superconductors where the mechanism for superconductivity is unknown. The amplitude mode of a superconductor was described theoretically within the Bardeen-Cooper-Schrieffer (BCS) model as the precession of pseudospins at a frequency given by twice the superconducting gap size [1], and has recently attracted attention after its experimental observation in a time-resolved THz-transmission measurement on the  $s$ -wave superconductor  $\text{Nb}_{0.8}\text{Ti}_{0.2}\text{N}$  [2]. Theoretical work on the Higgs mode has often been limited to performing a quantum quench of the pairing interaction which neglects important dynamical processes present in real materials such as melting of the superconducting order and is also limited to the framework of BCS theory which neglects inelastic scattering processes [3–7].

Since the Higgs mode is a scalar boson without charge or spin, it does not couple linearly to electromagnetic fields and is difficult to observe via the standard experimental probes of the equilibrium state [8, 9]. However, the emerging pump-probe technique of time- and angle-resolved photoemission spectroscopy (trARPES) is an ideal candidate for observing the Higgs mode because it is capable of exciting the Higgs mode and directly probing its presence by observing the behavior of the superconducting gap size on a femtosecond timescale at various momenta in the Brillouin zone [10]. The ultrashort pump pulse nonadiabatically excites the system to a nonequi-

librium state for which the original magnitude of the superconducting order parameter in the equilibrium state is no longer a minimum of the free energy. Because superconductivity is partially melted by the pump, the Higgs mode is then the oscillation of the superconducting order parameter about a new, smaller value due to the decrease in quasiparticles involved in ordering [4].

Recently, trARPES has successfully been used to study unoccupied bandstructure, relaxation dynamics, and collective modes in various materials, providing new information beyond the reach of equilibrium spectroscopies [11, 12, 14–21]. For instance, trARPES was used to directly probe the single-particle spectral function and observe the amplitude mode corresponding to oscillations of the charge-density wave order parameter [11]; such a measurement is possible due to the time-, energy-, and momentum-resolution of trARPES. The previous successes and future expectations for trARPES spectroscopy motivate us to perform theoretical simulations to complement this technique.

In this work, we go beyond the BCS model by including inelastic scattering processes (which are important for the dynamics out of equilibrium) and a frequency dependent pairing interaction. Within this framework, we investigate the characteristics of the Higgs mode for superconductors with  $d$ -wave pair symmetry which is relevant to the cuprate superconductors. One might have naively expected the Higgs mode in an anisotropic superconductor to oscillate at a frequency equal to twice the gap size at each given momentum  $k$ . Instead, we find that the Higgs mode appears as a phase-coherent oscillation in the trARPES spectra for a  $d$ -wave superconductor at a uniform frequency of twice the quasi-steady-state maximum gap size after pumping. This extends the previous work of Ref. [10] which studied the Higgs

mode for the case of an  $s$ -wave superconductor. In our calculations, we simulate the pump-probe process by self-consistently solving the Nambu-Gor'kov equations within the Migdal-Eliashberg approximation. Our calculation naturally captures the dynamic processes of melting of the superconducting state by the pump pulse and subsequent relaxation due to electron-boson scattering.

We solve the time-dependent equations of motion for the Holstein model with a momentum-dependent electron-boson coupling [22]

$$\mathcal{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} c_{\mathbf{k}} + \Omega \sum_{\mathbf{k}} b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} - \frac{1}{\sqrt{N}} \sum_{\mathbf{k}, \mathbf{q}} g(\mathbf{k}, \mathbf{q}) c_{\mathbf{k}-\mathbf{q}}^{\dagger} c_{\mathbf{k}} (b_{\mathbf{q}}^{\dagger} + b_{-\mathbf{q}}). \quad (1)$$

The trARPES spectrum is obtained from the double-time lesser Green's function on the Kadanoff-Baym-Keldysh contour as described in Ref. [23]. To calculate the double-time Green's functions on the contour, we self-consistently solve the Nambu-Gor'kov equations of motion. For a complete description of the method refer to the methods and model section of Ref. [10]. We use units where  $c = \hbar = e = 1$ . The coupling to the field is included to all orders via the Peierls substitution  $\mathbf{k}(t) = \mathbf{k} - \mathbf{A}(t)$  where  $\mathbf{A}(t)$  is the time varying vector potential in the Hamiltonian gauge. We ensure that the single-particle ARPES spectra are gauge invariant by performing the constructive transformation described by Ref. [24] which gauge shifts the momentum variable of the Green's function. The field of the pump pulse in all simulations below is applied along the diagonal direction of the Brillouin zone and takes the form of a sinusoidal oscillation (energy of 1.5 eV) with a Gaussian envelope (FWHM of 9.3 fs).

Superconductivity in our model is mediated through a generic bosonic mode which is included in the electron self-energy at self-consistent Born level. For this self-energy we take

$$\Sigma_{\mathbf{k}}^c(t, t') = \frac{i}{N_k} \sum_{\mathbf{k}'} |g(\mathbf{k}, \mathbf{k}')|^2 \tau_3 G_{\mathbf{k}'}^c(t, t') \tau_3 D_0^c(t, t'), \quad (2)$$

where  $D_0^c$  is the bare phonon propagator for an Einstein mode with frequency  $\Omega$ ,  $N_k$  is the number of momenta,  $\tau_3$  is the  $z$ -direction Pauli matrix in Nambu space, and the superscript  $c$  indicates contour-ordering on the Kadanoff-Baym-Keldysh contour. Generically,  $|g(\mathbf{k}, \mathbf{k}')|^2 = g_s + g_d d_{\mathbf{k}} d_{\mathbf{k}'}$  where  $g_s$  and  $g_d$  are constants reflecting the coupling strength in the  $s$ -wave and  $d$ -wave channels respectively,  $d_{\mathbf{k}} = \frac{1}{2}[\cos(k_x) - \cos(k_y)]$  is a momentum-dependent form-factor with  $d$ -wave symmetry, and  $\mathbf{k}' = \mathbf{k} - \mathbf{q}$ . We ensure a purely  $d$ -wave superconducting state by taking  $g_s = 0$  for the anomalous components of the Nambu self-energy, but we keep both  $s$ - and  $d$ -wave components in the diagonal part of the

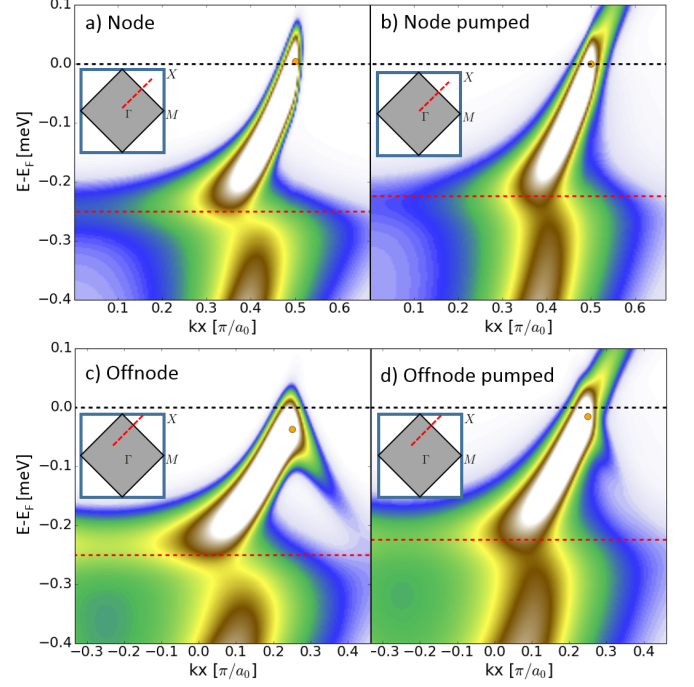


FIG. 1: **trARPES spectra.** In all panels, the orange marker indicates the peak position of the EDC at  $k_F$  which corresponds to the superconducting gap size, and the red dotted line indicates the energy of the bosonic mode plus the antinodal gap size which corresponds to the gap-shifted position of the kink in the bandstructure. a) Nodal ARPES spectrum in equilibrium with superconductivity. b) Nodal spectrum 25 fs after pump arrives. c) Off-nodal ARPES spectrum in equilibrium with superconductivity. d) Pumped off-nodal spectrum shows shift in the kink and a partial melting of the superconducting gap.

Nambu self-energy. We verify that the electron-boson interaction strength remains constant at all times (assuming the boson is unrenormalized and behaves as an infinite heat bath) by checking that the zeroth-moment of the retarded self-energy given by  $\Sigma^R(t, t)$  (which is proportional to the square of the coupling strength) is constant [25]. Therefore changes in the trARPES spectrum are not a result of changing the electron-boson coupling as in a quantum "quench", but instead a consequence of redistribution of spectral weight by the pump and transient effective electron-boson interactions which are determined self-consistently.

The equations of motion are solved by performing a massively parallel computation following the methods in Ref. [26]. For ease of simulation, we take a tight-binding model on a square lattice at half-filling with a nearest-neighbor hopping of  $V_{nn} = 0.25$  eV. We take a mode energy of  $\Omega = 0.2$  eV, a temperature of  $T \simeq 80$  K, and a coupling strength of  $g_s = g_d = \sqrt{0.12}$  eV which results in

a dimensionless coupling of  $\lambda_s \equiv -\partial \text{Re}\Sigma^R(\omega)/\partial \omega|_{\omega \rightarrow 0} = 0.67$ . The temperature is well below the transition temperature of  $T_c \simeq 240$  K.

In Fig. 1a, we show the ARPES spectrum in equilibrium near the Fermi level along a nodal cut (diagonal cut) which shows no gap at the Fermi level, as expected, and a kink in the bandstructure at the bosonic mode energy (200 meV) plus the maximum of the superconducting gap size (51 meV) [27, 29]. The superconducting gap size can be obtained from equilibrium self-energies or directly from the peak position of an antinodal energy-density curve (EDC) at  $k_F$ . For an off-nodal cut we take a cut parallel to the zone diagonal and halfway between the node and antinode. The spectrum for the off-nodal cut is shown in Fig. 1c and shows a gap at the Fermi level and a clear bend-back of the band due to particle-hole mixing. To determine the superconducting gap size, we find the peak position of the energy density curves (EDCs) at  $k = k_F$  by fitting to a Voigt profile. To determine the kink position, we use the energy of the inflection point in the Engelsberg-Schrieffer peak-dip-hump structure of the EDCs at  $k = k_F$  [27]. The gap position and kink positions are indicated with the colored markers and dotted lines in Fig. 1. We track these features as a function of time in the trARPES spectra. Figure 1(b,d) show the spectra 25 fs after the center of the pump pulse arrives. Spectral weight is redistributed above the Fermi level and the superconducting gap partially melts, also shifting the kink position to a higher binding energy.

After the pump pulse, clear oscillations occur in both the gap position and the kink position for various momenta along the Fermi surface between the node and the antinode as shown in Fig. 2. These oscillations are the amplitude (or Higgs) mode oscillations. Previous work has considered how the Higgs mode is affected by the continuum of single-particle excitations which exhibit a square-root singularity at  $2\Delta$  [28]. We note that the presence of oscillations at the kink position (roughly  $4\Delta$  away from the gap-edge) implies that it is not possible to attribute these oscillations to  $2\Delta$  quasiparticle excitations. Furthermore, the normal state spectra after pumping return monotonically to equilibrium, indicating that the superconducting order is responsible for the oscillations [10]. The oscillations of the gap become weaker and disappear towards the nodal point since the gap size shrinks to zero at the node. However, the value of the EDC maximum in Fig. 2b is not identically zero at the node because of broadening of the single-particle spectrum due to finite energy resolution.

The frequencies of the gap and kink oscillations are extracted by fitting to a decaying exponential plus a damped oscillation of the form:

$$Ae^{-t/\tau} + B \sin(\omega t + \phi)/t^p + D. \quad (3)$$

When used to fit the gap position, the parameter D gives

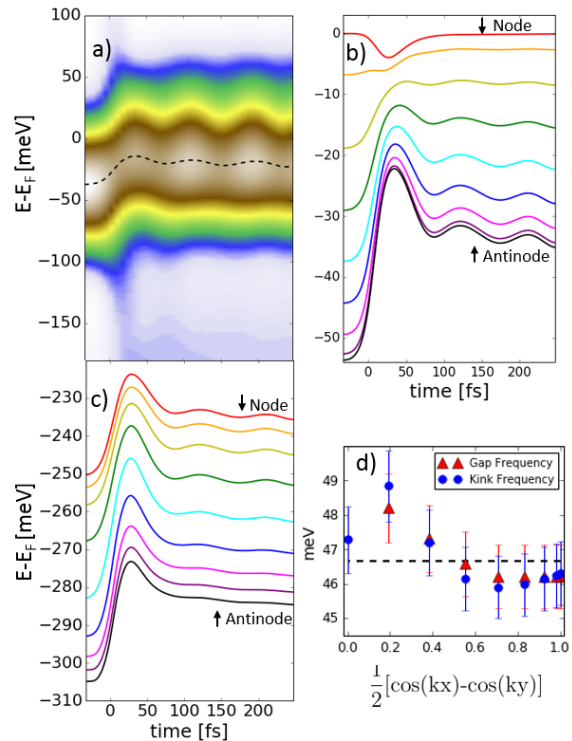


FIG. 2: **Gap and kink dynamics.** a) Tracking the peak position of the EDC at  $k_F$  given by the orange marker in Fig. 1 (i.e. superconducting gap size) for an off-nodal cut which is halfway between the node and antinode. b) Tracking the EDC peak position for multiple cuts along the Fermi surface. c) Tracking kink position given by the red dotted line in Fig. 1 based on the inflection point in the peak-dip-hump structure of EDCs at  $k_F$  for multiple cuts. Curves for the kink position are offset for clarity. The pump fluence is  $E_{max} = 1.2V/a_0$  for all plots. The time is measured relative to the center of the pump pulse (which reaches the sample at time  $t = 0$  fs). d) The frequency of the oscillations (of the gaps and the kinks) occurs at a single frequency given by the average value of 47 meV, shown as the dotted black line.

$\Delta_\infty$ , the quasi-steady-state value of the superconducting gap after the pump pulse (such that the Higgs frequency satisfies  $\omega = 2\Delta_\infty$ ). When used to fit the kink position, the parameter D is given by  $\Omega + \Delta_\infty$ . Our fits of the gap position do not extend all the way to the nodal point because as the gap value becomes smaller the oscillations decrease in amplitude and become more difficult to fit. However, the kink oscillations can still be fit at the node. From the combined analysis of both gap and kink position, we find that the Higgs oscillations occur at a single frequency and in phase across all momentum points within our frequency and energy resolution, as shown in Fig. 2d.

In Fig. 3, we again use the functional form in Eq. (3) to fit the EDC peak position at the antinode for different pump fluences. We observe that the Higgs oscillation

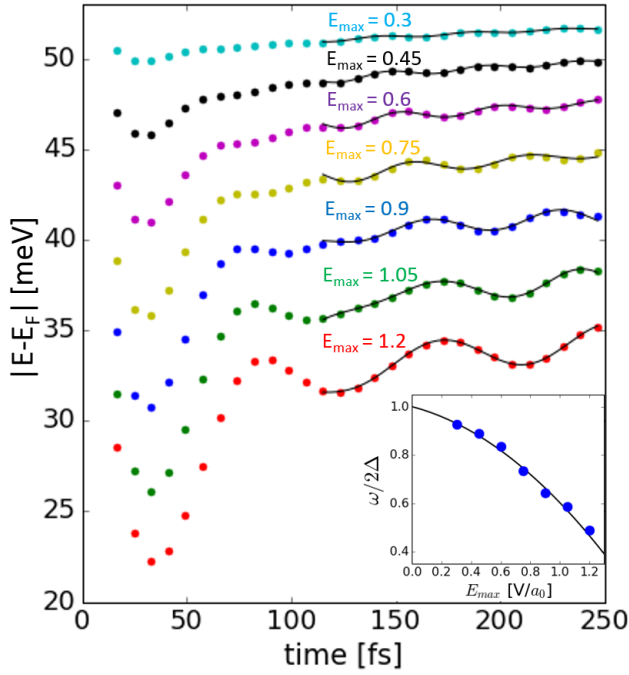


FIG. 3: **Gap dynamics vs. fluence.** The superconducting gap size (determined by the magnitude of the antinodal EDC peak position) as a function of time for different pump fluences (maximum electric field in  $V/a_0$ ). Solid lines show the fits. Inset: in the zero fluence limit, the Higgs frequency extrapolates to twice the maximum gap size in equilibrium. The solid line is a quadratic polynomial fit.

frequency decreases with increasing fluence because the superconducting gap size is suppressed more for stronger pumping [10]. The frequency of the Higgs oscillation follows twice the quasi-steady state value of the antinodal (maximum) gap size after pumping ( $\Delta_\infty$ ), and the Higgs frequency extrapolates to twice the value of the antinodal superconducting gap size in equilibrium as shown in the inset in Fig. 3. In other words, for a  $d$ -wave superconductor, the Higgs mode is a  $2\Delta_\infty$  oscillation with  $\Delta_\infty$  given by the maximum gap size after pumping.

The Higgs mode has yet to be detected in trARPES experiments which have up to this point mainly focused on relaxation dynamics of quasiparticles and other collective modes [11–21, 32]. In order to satisfy the experimental conditions necessary for observing the Higgs mode, the fluence of the pump pulse must be tuned and both the pump and the probe pulse must be sufficiently fast. Naturally, if the pump is too weak, the amplitude of the Higgs mode will be small. If the pump is too strong, the condensate is fully depleted and Cooper pairs are not available to participate in the collective mode. From our simulations, approximately depleting half of the condensate results in the strongest Higgs oscillations. Recent experiments have been able to produce a pump pulse with sufficient fluence (on the order of  $10\mu\text{J}/\text{cm}^2$ )

to completely destroy the condensate [19, 32], so tuning to a lower fluence should be possible. To determine the required width of the pump and probe pulses, we must consider the timescale of the Higgs mode which is set by  $\tau = \hbar/2\Delta$ . For the pump pulse to nonadiabatically excite the condensate, the width of the pump pulse must be less than  $\tau$ . In addition, for the probe pulse to resolve the oscillations, the width of the probe pulse must also be less than  $\tau$ . For a typical cuprate superconductor such as  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$  around optimal doping, the equilibrium superconducting gap size is of the order of 40 meV in the region around optimal doping [31]. If the gap is suppressed by 50 percent after pumping, the timescale of the Higgs mode will be on the order of  $\tau = 200$  fs. It is promising that some experimental groups have achieved a resolution of around 100 fs [12, 13]. A separate factor which could potentially prevent the detection of the Higgs mode in some systems is the presence of inter-band transitions which could destroy the coherent nature of the collective mode. How the Higgs mode appears in multi-band systems with dipole transitions should be clarified in the future.

The stage is set to take advantage of pump-probe techniques such as trARPES not only to detect the Higgs mode, but also to study the rich assortment of collective modes which have been predicted in the unconventional superconductors. Examples include the Bardasis-Schrieffer modes in systems with competition for superconducting ground states with different pairing symmetries [34], Leggett modes in multi-band superconductors [35, 36], multiple Higgs modes in channels corresponding to different irreducible representations of the lattice [3], and the various collective modes arising in gauge theories [37]. Our work serves as a starting point for studying these modes within a framework that includes inelastic scattering and retarded interactions, ingredients which are needed to accurately simulate the amplitude mode in a superconductor out of equilibrium during a pump-probe experiment. For a  $d$ -wave superconductor we find a Higgs mode at a single frequency equal to twice the maximum renormalized gap size. The parameters chosen in the simulation for the pump and probe pulses are already feasible in current trARPES setups. Under these conditions, we predict that the Higgs mode can be detected in trARPES experiments as oscillations in spectral intensity at the gap edge as well as down to the energy of the pair boson.

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- [1] P. W. Anderson, Phys. Rev. Lett. **112** 1900 (1958).
  - [2] R. Matsunaga, N. Tsuji, H. Fujita, A. Sugioka, K. Makise, Y. Uzawa, H. Terai, Z. Wang, H. Aoki, and R. Shimano, Science **345**, 1145 (2014).
  - [3] Y. Barlas and C. M. Varma, Phys. Rev. B **87**, 054503 (2013).
  - [4] A. F. Volkov and S. M. Kogan, Soviet Journal of Experimental and Theoretical Physics **38**, 1018 (1974).
  - [5] E. A. Yuzbashyan, O. Tsypliyatyev, and B. L. Altshuler, Phys. Rev. Lett. **96**, 097005 (2006).
  - [6] N. Tsuji and Hideo Aoki, Phys. Rev. B **92**, 064508 (2015).
  - [7] F. Peronaci, M. Schiro, and M. Capone, Phys. Rev. Lett. **115**, 257001 (2015).
  - [8] D. Pekker and C. M. Varma, Annual Review of Condensed Matter Physics **6**, 269 (2015).
  - [9] D. Podolsky, A. Auerbach, and D. P. Arovas, Phys. Rev. B **84**, 174522 (2011).
  - [10] A. F. Kemper, M. A. Sentef, B. Moritz, J. K. Freericks, and T. P. Devereaux, Phys. Rev. B **92**, 224517 (2015).
  - [11] F. Schmitt, P. S. Kirchmann, U. Bovensiepen, R. G. Moore, L. Rettig, M. Krenz, J.-H. Chu, N. ru, L. Perfetti, D. H. Lu, M. Wolf, I. R. Fisher, and Z.-X. Shen, Science **321**, 1649 (2008).
  - [12] S.-L. Yang, J. A. Sobota, D. Leuenberger, Y. He, M. Hashimoto, D. H. Lu, H. Eisaki, P. S. Kirchmann, and Z.-X. Shen, Phys. Rev. Lett. **114**, 247001 (2015).
  - [13] S.-L. Yang, J. A. Sobota, D. Leuenberger, A. F. Kemper, J. J. Lee, F. T. Schmitt, W. Li, R. G. Moore, P. S. Kirchmann, and Z.-X. Shen, Nano Lett. **15**, 4150 (2015).
  - [14] J. Graf, C. Jozwiak, C. L. Smallwood, H. Eisaki, R. A. Kaindl, D.-H. Lee, and A. Lanzara, Nature Physics **7**, 805-809 (2011).
  - [15] C. L. Smallwood, J. P. Hinton, C. Jozwiak, W. Zhang, J. D. Koralek, H. Eisadi, D.-H. Lee, J. Orenstein, and A. Lanzara, Science **336**, 1137 (2012).
  - [16] J. D. Rameau, S. Freutel, M. A. Sentef, A. F. Kemper, J. K. Freericks, I. Avigo, M. Ligges, L. Rettig, Y. Yoshida, H. Eisaki, J. Schneeloch, R. D. Zhong, Z. J. Xu, G. D. Gu, P. D. Johnson, and U. Bovensiepen, ArXiv e-prints (2015), arXiv: 1505.07055 [cond-mat.supr-con].
  - [17] J. D. Rameau, S. Freutel, L. Rettig, I. Avigo, M. Ligges, Y. Yoshida, H. Eisaki, J. Schneeloch, R. D. Zhong, Z. J. Xu, G. D. Gu, P. D. Johnson, and U. Bovensiepen, Phys. Rev. B **89**, 115115 (2014).
  - [18] I. Avigo, S. Thirupathaiah, M. Ligges, T. Wolf, J. Fink, U. Bovensiepen, ArXiv e-prints (2016), arXiv: 1605.05177v1 [cond-mat.supr-con].
  - [19] C. Piovera, Z. Zhang, M. d'Astuto, A. Taleb-Ibrahimi, E. Paplazarou, M. Marsi, Z. Z. Li, H. Raffy, and L. Perfetti, Phys. Rev. B **91**, 224509 (2015).
  - [20] S. Zhu, Y. Ishida, K. Kuroda, K. Sumida, M. Ye, J. Wang, H. Pan, M. Taniguchi, S. Qiao, S. Shin, and A. Kimura, Sci. Rep. **5**, 13213 (2015).
  - [21] J. C. Peterson, S. Kaiser, N. Dean, A. Simoncig, H. Y. Liu, A. L. Cavalieri, C. Cacho, I. C. E. Turcu, E. Springate, F. Frassetto, L. Poletto, S. S. Dhesi, H. Berger, and A. Cavalleri, Phys. Rev. Lett. **107**, 177402 (2011).
  - [22] T. Holstein, Ann. Phys. **8**, 325 (1959).
  - [23] J. K. Freericks, H. R. Krishnamurthy, and T. Pruschke, Phys. Rev. Lett. **102**, 136401 (2009).
  - [24] R. Bertoncini and A. P. Jauho, Phys. Rev. B **44**, 3655 (1991).
  - [25] A. F. Kemper, M. A. Sentef, B. Moritz, J. K. Freericks, and T. P. Devereaux, Phys. Rev. B **90**, 075126 (2014).
  - [26] G. Stefanucci and R. van Leeuwen, *Nonequilibrium Many-body Theory of Quantum Systems: A Modern Introduction* (Cambridge University Press, New York, 2013).
  - [27] A. W. Sandvik, D. J. Scalapino, and N. E. Bickers, Phys. Rev. B **69**, 094523 (2004).
  - [28] T. Cea and L. Benfatto, Phys. Rev. B **90**, 224515 (2014).
  - [29] T. Cuk, D. H. Lu, X. J. Zhou, Z.-X. Shen, T. P. Devereaux, and N. Nagaosa, Phys. Stat. Sol. (b) **242**, 11 (2005).
  - [30] Y. Murakami, P. Werner, N. Tsuji, and H. Aoki, ArXiv e-prints (2016), arXiv: 1604.08073v1 [cond-mat.supr-con].
  - [31] I. M. Vishik, M. Hashimoto, R.-H. He, W. S. Lee, F. Schmitt, D. H. Lu, R. G. Moore, C. Zhang, W. Meevasana, T. Sasagawa, S. Uchida, K. Fujita, S. Ishida, M. Ishikado, Y. Yoshida, H. Eisaki, Z. Hussain, T. P. Devereaux, and Z.-X. Shen, PNAS **109**, 18332 (2012).
  - [32] W. Zhang, T. Miller, C. L. Smallwood, Y. Yoshida, H. Eisaki, R. A. Kaindl, D.-H. Lee, and A. Lanzara, Sci. Rep. **6**, 29100 (2016).
  - [33] C. L. Smallwood, W. Zhang, T. L. Miller, C. Jozwiak, H. Eisaki, D.-H. Lee, and A. Lanzara, Phys. Rev. B **89**, 115126 (2014).
  - [34] T. Bohm, A. F. Kemper, B. Moritz, F. Kretzschmar, B. Muschler, H.-M. Eiter, R. Hackl, T. P. Devereaux, D. J. Scalapino, and H.-H. Wen, Phys. Rev. X **4**, 041046 (2014).
  - [35] H. Krull, N. Bittner, G. S. Uhrig, D. Manske, and A. P. Schnyder, Nature Communications **7**, 11921 (2016).
  - [36] W. Huang, T. Scaffidi, M. Sigrist, and C. Kallin, ArXiv e-prints (2016), arXiv: 1605.03800v2 [cond-mat.supr-con].
  - [37] P. A. Lee, N. Nagaosa, Phys. Rev. B **68**, 024516 (2003).